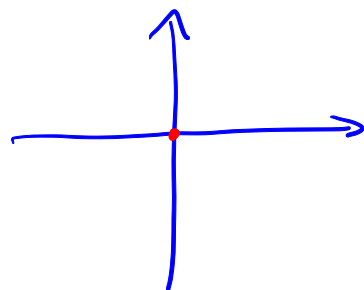


Visualizing Span

Example 2: Sketch the following sets in \mathbb{R}^2 . Give a geometric description of each set.

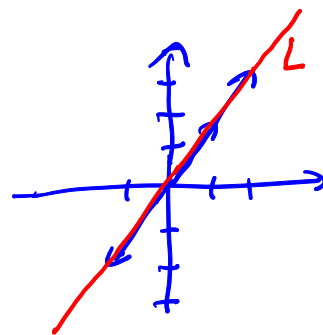
1. $\text{span} \left(\begin{bmatrix} 0 \\ 0 \end{bmatrix} \right) = \left\{ t \begin{bmatrix} 0 \\ 0 \end{bmatrix} : t \text{ in } \mathbb{R} \right\} = \{ \vec{0} \}$

A point in \mathbb{R}^2 (origin)



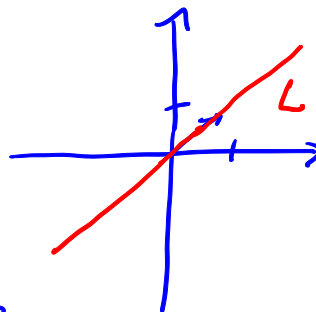
2. $\text{span} \left(\begin{bmatrix} 1 \\ 2 \end{bmatrix} \right) = \left\{ t \begin{bmatrix} 1 \\ 2 \end{bmatrix} : t \text{ in } \mathbb{R} \right\}$

A line in \mathbb{R}^2 containing the origin.



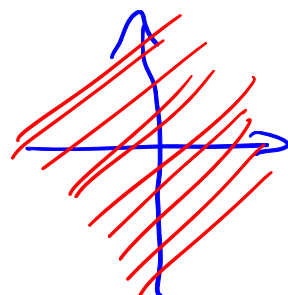
3. $\text{span} \left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \end{bmatrix} \right) = \left\{ s \begin{bmatrix} 1 \\ 1 \end{bmatrix} + t \begin{bmatrix} 2 \\ 2 \end{bmatrix} : s \text{ and } t \text{ in } \mathbb{R} \right\}$
 $= \left\{ (s + 2t) \begin{bmatrix} 1 \\ 1 \end{bmatrix} : s \text{ and } t \text{ in } \mathbb{R} \right\} = \text{span} \left(\begin{bmatrix} 1 \\ 1 \end{bmatrix} \right)$

A line in \mathbb{R}^2 containing the origin.



4. $\text{span} \left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right) = \left\{ s \begin{bmatrix} 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} 0 \\ 1 \end{bmatrix} : s \text{ and } t \text{ in } \mathbb{R} \right\}$
 $= \left\{ \begin{bmatrix} s \\ t \end{bmatrix} : s \text{ and } t \text{ in } \mathbb{R} \right\} = \mathbb{R}^2$

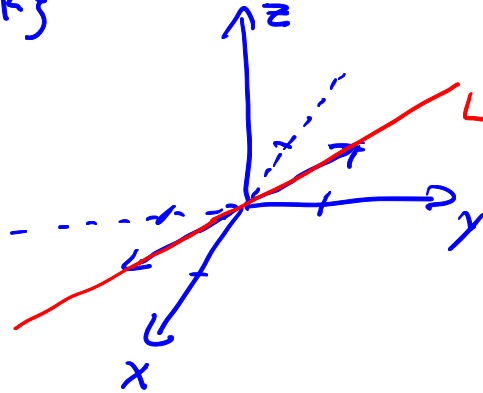
All of \mathbb{R}^2



Example 3: Sketch the following sets in \mathbb{R}^3 . Give a geometric description of each set.

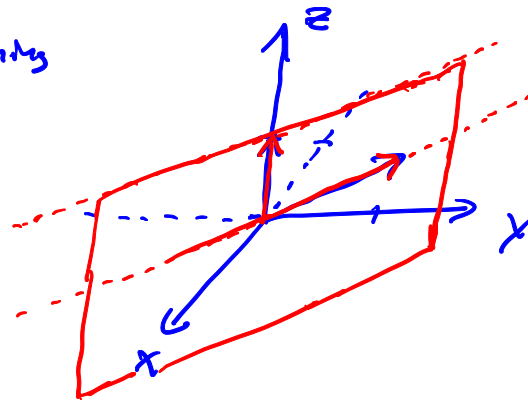
$$1. \text{span} \left(\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} \right) = \left\{ t \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} : t \text{ in } \mathbb{R} \right\}$$

A line in \mathbb{R}^3 containing the origin.



$$2. \text{span} \left(\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right) = \left\{ s \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} : s \text{ and } t \text{ in } \mathbb{R} \right\}$$

A plane in \mathbb{R}^3 containing the origin.



$$3. \text{span} \left(\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right) = \left\{ r \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + s \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} : r, s, t \text{ in } \mathbb{R} \right\}$$

$$= \left\{ \begin{bmatrix} r \\ s \\ t \end{bmatrix} : r, s, t \text{ in } \mathbb{R} \right\} = \mathbb{R}^3$$

all of \mathbb{R}^3

